

P2-12-2-8

403648

CODE

8

## PAPER 2

Time: 3 Hours

Maximum Marks: 198

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

**INSTRUCTIONS:****A. General**

1. This booklet is your Question Paper. Do not break the seals of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the right hand top corner of this page and on the back page (Page No. 36) of this booklet.
3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.
5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of the examination. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
6. Using a black ball point pen, darken the bubbles on the upper original sheet. Apply sufficient pressure so that the impression is created on the bottom sheet.
7. DO NOT TAMPER WITH/MUTILATE THE ORS OR THE BOOKLET.
8. On breaking the seals of the booklet check that it contains 36 pages and all the 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

**B. Filling the Right Part of the ORS**

9. The ORS has CODES printed on its left and right parts.
10. Check that the same CODE is printed on the ORS and on this booklet. IF IT IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET. Sign at the place provided on the ORS affirming that you have verified that all the codes are same.
11. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. Do not write any of this information anywhere else. Darken the appropriate bubble UNDER each digit of your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of ORS (R4).

**C. Question Paper Format**

The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part consists of three sections.

12. Section I contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.
13. Section II contains 3 paragraphs each describing theory, experiment, data etc. There are 6 multiple choice questions relating to three paragraphs with 2 questions on each paragraph. Each question of a particular paragraph has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.
14. Section III contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

**D. Marking Scheme**

15. For each question in Section I and Section II, you will be awarded 3 marks if you darken the bubble corresponding to the correct answer ONLY and zero (0) marks if no bubbles are darkened. In all other cases, minus one (-1) mark will be awarded in these sections.
16. For each question in Section III, you will be awarded 4 marks if you darken ALL the bubble(s) corresponding to the correct answer(s) ONLY. In all other cases zero (0) marks will be awarded. No negative marks will be awarded for incorrect answer(s) in this section.

DO NOT BREAK THE SEALS WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

SEAL

SEAL

Write Your Name, Registration Number and sign in the space provided on the back page of this booklet.



	Subject	Section		018801
Part I	Physics	I	Single Correct Answer Type	3 – 6
		II	Paragraph Type	7 – 11
		III	Multiple Correct Answer(s) Type	12 – 14
Part II	Chemistry	I	Single Correct Answer Type	15 – 17
		II	Paragraph Type	18 – 20
		III	Multiple Correct Answer(s) Type	21 – 25
Part III	Mathematics	I	Single Correct Answer Type	26 – 27
		II	Paragraph Type	28 – 30
		III	Multiple Correct Answer(s) Type	31 – 33

Space for Rough Work



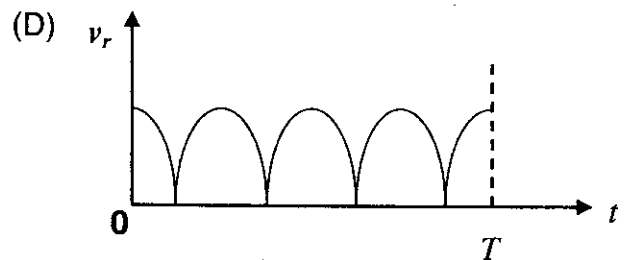
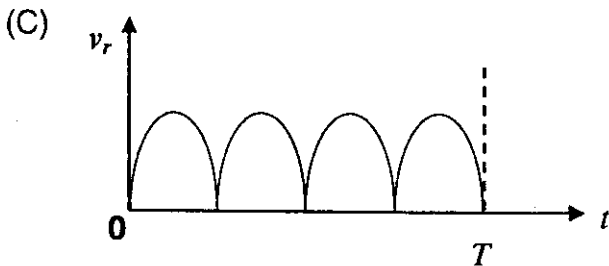
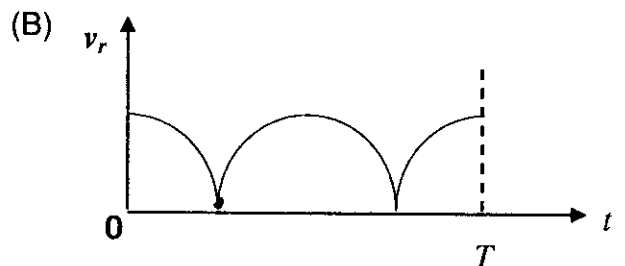
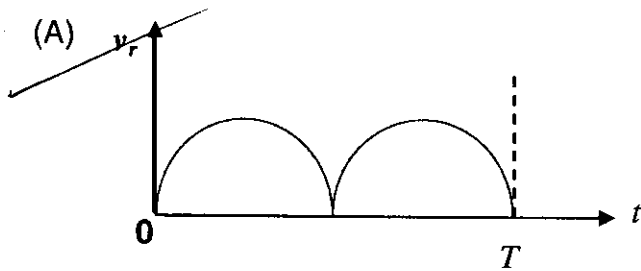
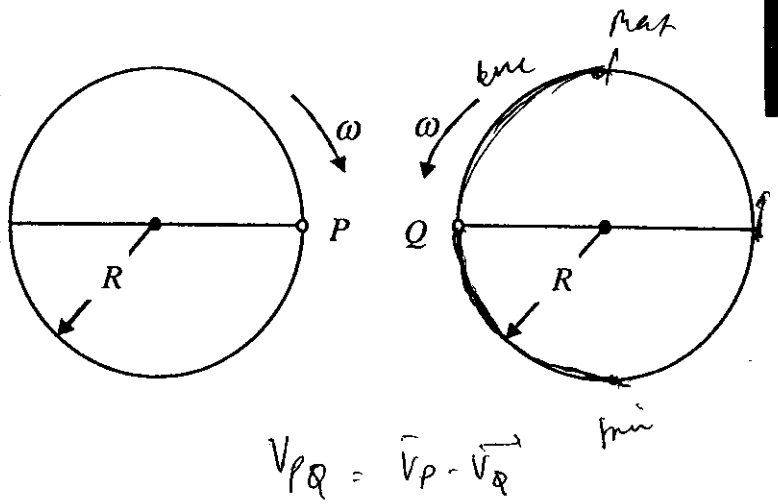
# PART I : PHYSICS

## SECTION I : Single Correct Answer Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

PHYSICS

1. Two identical discs of same radius  $R$  are rotating about their axes in opposite directions with the same constant angular speed  $\omega$ . The discs are in the same horizontal plane. At time  $t=0$ , the points  $P$  and  $Q$  are facing each other as shown in the figure. The relative speed between the two points  $P$  and  $Q$  is  $v_r$ . In one time period ( $T$ ) of rotation of the discs,  $v_r$  as a function of time is best represented by



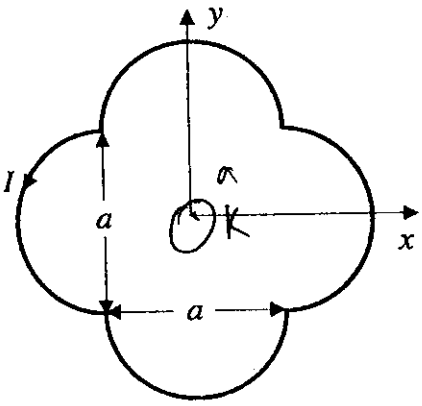
Space for Rough Work

*dec*



$(a^2 + 2Aa^2)$   
 $\frac{2\pi a^2 (2a)}{NiA} \rightarrow AB Ni \sin \theta$   
 $a^2 (1+2\pi) I$

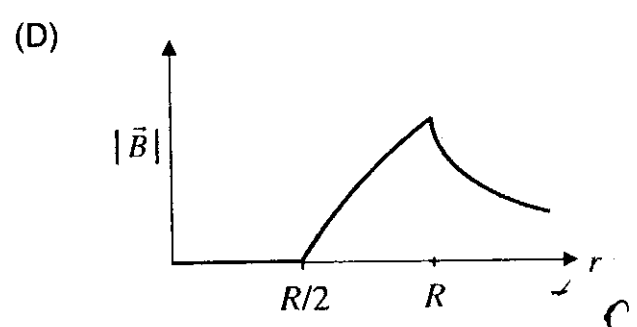
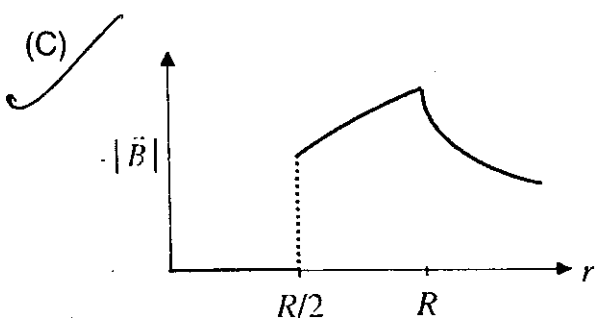
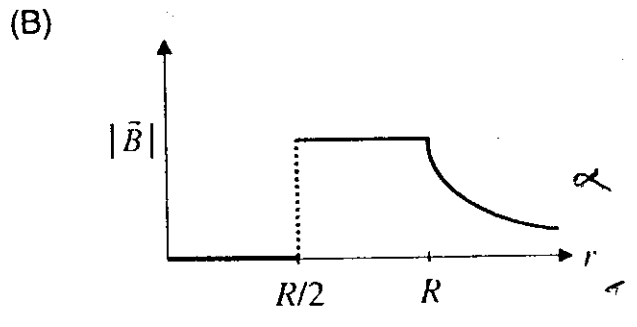
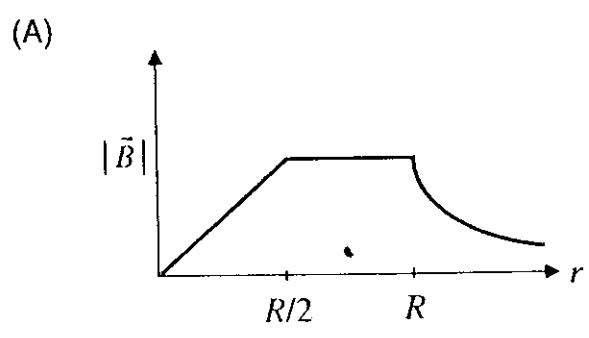
2. A loop carrying current  $I$  lies in the  $x$ - $y$  plane as shown in the figure. The unit vector  $\hat{k}$  is coming out of the plane of the paper. The magnetic moment of the current loop is



$\frac{C_p}{C_v} = \gamma$        $C_p - C_v = R$   
 $C_v = \frac{1}{2} fR$        $C_p = R + C_v$   
 $= \frac{3}{2} R$        $= R + \frac{3}{2} R$   
 $\gamma = \frac{\frac{5}{2} R}{\frac{3}{2} R} = \frac{5}{3}$

- (A)  $a^2 I \hat{k}$       (B)  $(\frac{\pi}{2} + 1) a^2 I \hat{k}$       (C)  $-(\frac{\pi}{2} + 1) a^2 I \hat{k}$       (D)  $(2\pi + 1) a^2 I \hat{k}$

3. An infinitely long hollow conducting cylinder with inner radius  $R/2$  and outer radius  $R$  carries a uniform current density along its length. The magnitude of the magnetic field,  $|\vec{B}|$  as a function of the radial distance  $r$  from the axis is best represented by



Space for Rough Work

$w = \frac{\mu R}{\gamma - 1} (T_1 - T_2)$   
 $= \frac{2(8.31) \cdot (1)}{2/3 - 1} = \frac{16.62}{-1/3} = -49.86$   
 $124.6 \text{ J}$

$$\frac{24\pi R^2}{2\pi R^2}$$

$$\frac{C_v}{C_R} = \gamma$$

$$C_v = \frac{1}{2} f R = \frac{1}{2} \cdot 3R$$

$$C_v = \frac{3}{2} R$$

$$C_p - C_v = R \quad C_p = C_v + R = \frac{5}{2} R$$

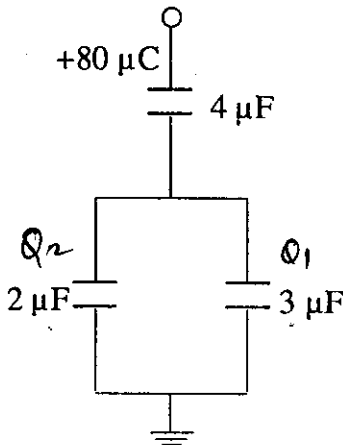
PHYSICS

4. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If  $\rho_c$  is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

- (A) more than half-filled if  $\rho_c$  is less than 0.5.
- (B) more than half-filled if  $\rho_c$  is more than 1.0.
- (C) half-filled if  $\rho_c$  is more than 0.5.
- (D) less than half-filled if  $\rho_c$  is less than 0.5.



5. In the given circuit, a charge of  $+80 \mu\text{C}$  is given to the upper plate of the  $4 \mu\text{F}$  capacitor. Then in the steady state, the charge on the upper plate of the  $3 \mu\text{F}$  capacitor is



(A)  $+32 \mu\text{C}$

(B)  $+40 \mu\text{C}$

(C)  $+48 \mu\text{C}$

(D)  $+80 \mu\text{C}$

~~Q2~~

$$Q = CV$$

$$B. 4\pi R^2 = \frac{\mu_0 3\pi R^2}{\mu_0 \pi (4R^2 - R^2)}$$

~~RAGO~~

$$\frac{Q_1}{3} = \frac{Q_2}{2}$$

$$2Q_1 = 3Q_2$$

$$Q_1 = 3Q_2 \quad B. 2\pi R = \frac{\mu_0 3\pi R^2}{\mu_0 \pi (4R^2 - R^2)}$$

$$\begin{aligned} 4R^2 - R^2 &= 3R^2 \\ 4R^2 &= 3R^2 \\ 2R &= R \\ 8 \cdot \frac{R}{2} & \end{aligned}$$

6. Two moles of ideal helium gas are in a rubber balloon at  $30^\circ\text{C}$ . The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to  $35^\circ\text{C}$ . The amount of heat required in raising the temperature is nearly (take  $R = 8.31 \text{ J/mol.K}$ )

(A) 62 J

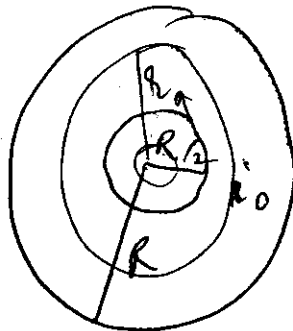
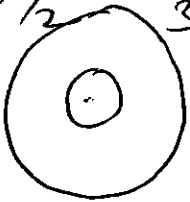
(B) 104 J

(C) 124 J

(D) 208 J

Space for Rough Work

$$\frac{Q_1}{Q_2} = \frac{3}{2} = \frac{48}{32}$$



so if  $R > R$

$$\mu = \frac{i}{16}$$

$$\mu = \frac{i}{4\pi R^2}$$

$$\mu = \frac{i_0}{4\pi R^2 - 4\pi \left(\frac{R^2}{4}\right)}$$

$$S = 4\pi R^2 - 4\pi R^2 = 2(4R^2 - R^2)$$

$$\mu = \frac{i_0}{3\pi R^2}$$

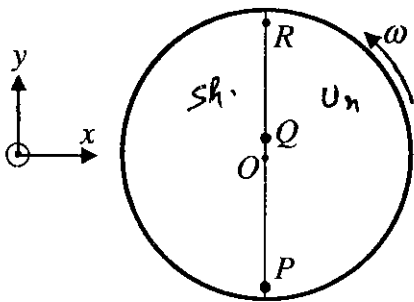
$$i = \frac{i_0 3(4R^2 - R^2)}{3\pi R^2}$$

$$\int \vec{B} \cdot d\vec{A} = \frac{\mu_0}{i}$$

\*\*8



7. Consider a disc rotating in the horizontal plane with a constant angular speed  $\omega$  about its centre  $O$ . The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles  $P$  and  $Q$  are simultaneously projected at an angle towards  $R$ . The velocity of projection is in the  $y$ - $z$  plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed  $\frac{1}{8}$  rotation, (ii) their range is less than half the disc radius, and (iii)  $\omega$  remains constant throughout. Then



- (A)  $P$  lands in the shaded region and  $Q$  in the unshaded region.
- (B)  $P$  lands in the unshaded region and  $Q$  in the shaded region.
- (C) Both  $P$  and  $Q$  land in the unshaded region.
- (D) Both  $P$  and  $Q$  land in the shaded region.

8. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is  $38^\circ\text{C}$  in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

- (A) 14.0 cm
- (B) 15.2 cm
- (C) 16.4 cm
- (D) 17.6 cm

Space for Rough Work

Handwritten work for question 8:

$336 \text{ m/s}$

$512 =$

$\lambda/4 = x$

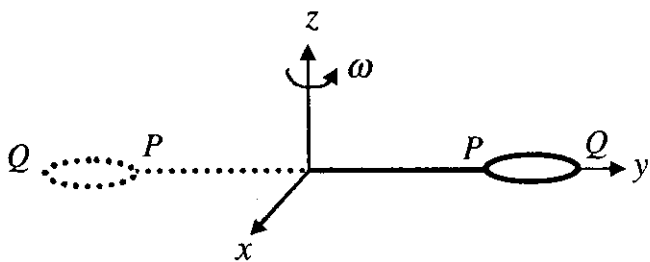


SECTION II : Paragraph Type

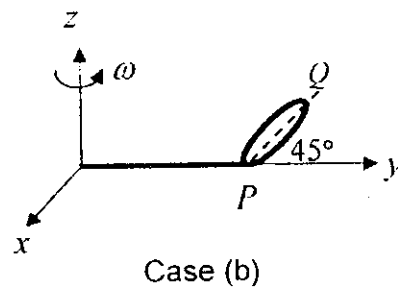
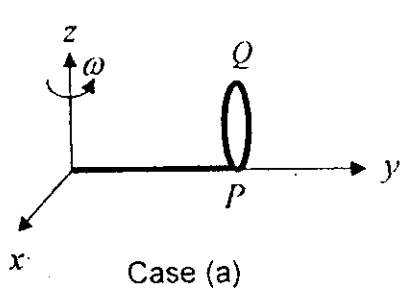
This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 9 and 10

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed  $\omega$ , the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed  $\omega$  in this case.



Now consider two similar systems as shown in the figure: Case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed  $\omega$  about the z-axis.



Handwritten calculation:  $\frac{128}{6} = 21.33$

Space for Rough Work

Handwritten calculations:  $\frac{84 \times 21}{336} = x$ ,  $4 \times 21 = 84$ ,  $8 \times 128$

Handwritten equations:  $\lambda = 4\pi$ ,  $\frac{v}{\lambda} = 4\pi$

Handwritten calculations:  $\frac{v}{4\pi} = x$ ,  $\frac{84 \times 21}{336} = x$ ,  $4 \times 21 = 84$ ,  $8 \times 128$

Handwritten equations:  $v = \lambda \omega$ ,  $\lambda = \frac{v}{\omega}$

Handwritten calculation:  $128 \overline{) 210}$ ,  $\frac{128}{820}$ ,  $\frac{767}{867}$ , Page No. 76



9. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?
- (A) It is vertical for both the cases (a) and (b).
  - (B) It is vertical for case (a); and is at  $45^\circ$  to the  $x$ - $z$  plane and lies in the plane of the disc for case (b).
  - (C) It is horizontal for case (a); and is at  $45^\circ$  to the  $x$ - $z$  plane and is normal to the plane of the disc for case (b).
  - (D) It is vertical for case (a); and is at  $45^\circ$  to the  $x$ - $z$  plane and is normal to the plane of the disc for case (b).
10. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?
- (A) It is  $\sqrt{2}\omega$  for both the cases.
  - (B) It is  $\omega$  for case (a); and  $\frac{\omega}{\sqrt{2}}$  for case (b).
  - (C) It is  $\omega$  for case (a); and  $\sqrt{2}\omega$  for case (b).
  - (D) It is  $\omega$  for both the cases.

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Space for Rough Work





Paragraph for Questions 11 and 12

The  $\beta$ -decay process, discovered around 1900, is basically the decay of a neutron ( $n$ ). In the laboratory, a proton ( $p$ ) and an electron ( $e^-$ ) are observed as the decay products of the neutron. Therefore, ~~considering the decay of a neutron as a two-body decay process, it was predicted~~ theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e.  $n \rightarrow p + e^- + \bar{\nu}_e$ , around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ( $\bar{\nu}_e$ ) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is  $0.8 \times 10^6$  eV. The kinetic energy carried by the proton is only the recoil energy.

11. If the anti-neutrino had a mass of  $3 \text{ eV}/c^2$  (where  $c$  is the speed of light) instead of zero mass, what should be the range of the kinetic energy,  $K$ , of the electron?
- (A)  $0 \leq K \leq 0.8 \times 10^6 \text{ eV}$                       (B)  $3.0 \text{ eV} \leq K \leq 0.8 \times 10^6 \text{ eV}$   
 (C)  $3.0 \text{ eV} \leq K < 0.8 \times 10^6 \text{ eV}$                       (D)  $0 \leq K < 0.8 \times 10^6 \text{ eV}$
12. What is the maximum energy of the anti-neutrino?
- (A) Zero.    (B) Much less than  $0.8 \times 10^6 \text{ eV}$ .  
 (C) Nearly  $0.8 \times 10^6 \text{ eV}$ .                      (D) Much larger than  $0.8 \times 10^6 \text{ eV}$ .

Space for Rough Work

$$n \rightarrow p + e^-$$

$$K = \frac{1}{2} \cdot 3 \times 10^8$$

$$K = 3 \text{ eV}$$



Paragraph for Questions 13 and 14

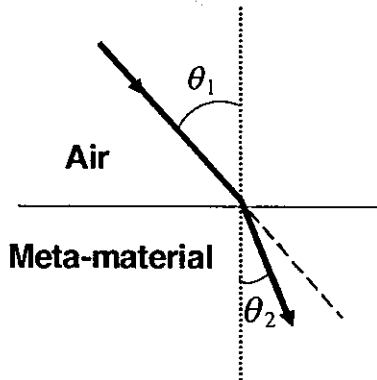
Most materials have the refractive index,  $n > 1$ . So, when a light ray from air enters a naturally occurring material, then by Snell's law,  $\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$ , it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation,

$n = \left(\frac{c}{v}\right) = \pm \sqrt{\epsilon_r \mu_r}$ , where  $c$  is the speed of electromagnetic waves in vacuum,  $v$  its speed in the medium,  $\epsilon_r$  and  $\mu_r$  are the relative permittivity and permeability of the medium respectively.

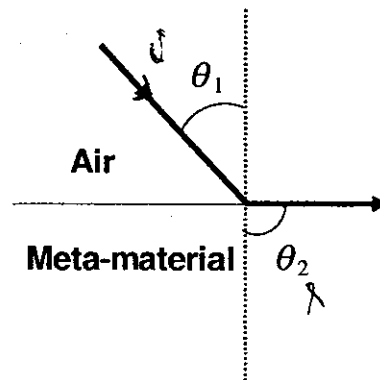
In normal materials, both  $\epsilon_r$  and  $\mu_r$  are positive, implying positive  $n$  for the medium. When both  $\epsilon_r$  and  $\mu_r$  are negative, one must choose the negative root of  $n$ . Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since  $n$  is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

13. For light incident from air on a meta-material, the appropriate ray diagram is

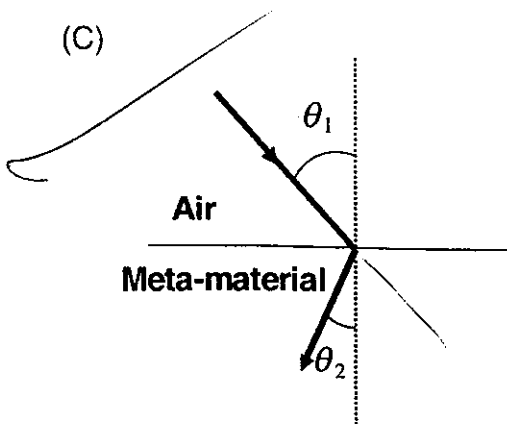
(A)



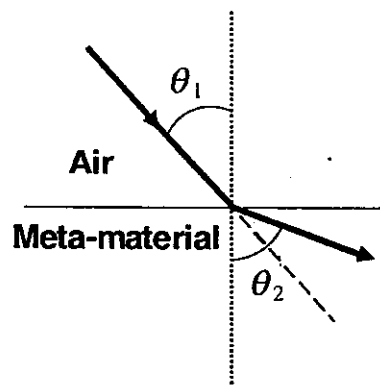
(B)



(C)



(D)



Space for Rough Work



14. Choose the correct statement.

- (A) The speed of light in the meta-material is  $v = c|n|$
- (B) The speed of light in the meta-material is  $v = \frac{c}{|n|}$
- (C) The speed of light in the meta-material is  $v = c$ .
- (D) The wavelength of the light in the meta-material ( $\lambda_m$ ) is given by  $\lambda_m = \lambda_{air}|n|$ , where  $\lambda_{air}$  is the wavelength of the light in air.

Space for Rough Work

$$n = \frac{c}{v}$$

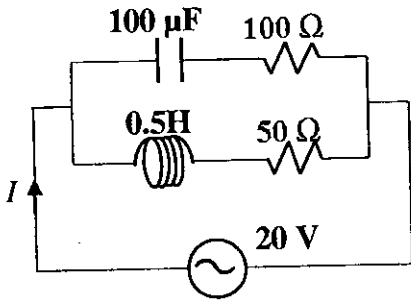
$$n = \frac{\lambda_{air}}{\lambda_m} \Rightarrow \lambda_m = \frac{\lambda_{air}}{n}$$



SECTION III : Multiple Correct Answer(s) Type

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

15. In the given circuit, the AC source has  $\omega = 100 \text{ rad/s}$ . Considering the inductor and capacitor to be ideal, the correct choice(s) is(are)



Handwritten notes for question 15:

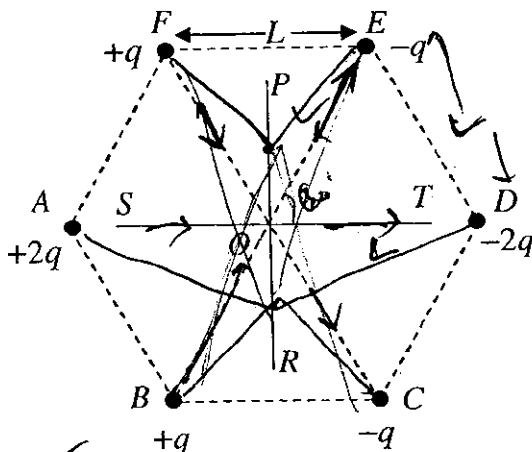
$$M = \frac{L}{v}$$

$$M = \rho \cdot v$$

$$\frac{4}{3} \pi \left( \frac{\sqrt{A}}{2} \right)^3 + \frac{4}{3} \pi \left( \frac{\sqrt{A}}{4a} \right)^3 = \dots$$

$$\frac{1}{3} \left[ \left( \frac{\sqrt{A}}{2} \right)^3 + \left( \frac{\sqrt{A}}{4a} \right)^3 \right] = \dots$$

- (A) The current through the circuit,  $I$  is  $0.3 \text{ A}$ .  
 (B) The current through the circuit,  $I$  is  $0.3\sqrt{2} \text{ A}$ .  
 (C) The voltage across  $100 \Omega$  resistor =  $10\sqrt{2} \text{ V}$ .  
 (D) The voltage across  $50 \Omega$  resistor =  $10 \text{ V}$ .
16. Six point charges are kept at the vertices of a regular hexagon of side  $L$  and centre  $O$ , as shown in the figure. Given that  $K = \frac{1}{4\pi\epsilon_0} \frac{q}{L^2}$ , which of the following statement(s) is(are) correct?



Handwritten notes for question 16:

$$E = \left( \frac{1}{4\pi\epsilon_0} \frac{q}{L^2} \right)^2 = \frac{1}{4\pi\epsilon_0} \frac{2q}{L^2}$$

Other notes:  $2K, 4K, 6K$

- (A) The electric field at  $O$  is  $6K$  along  $OD$ .  
 (B) The potential at  $O$  is zero.  
 (C) The potential at all points on the line  $PR$  is same.  
 (D) The potential at all points on the line  $ST$  is same.

Handwritten notes for question 16:

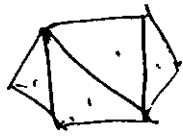
$$M = \frac{4}{3} \pi \left( \frac{\sqrt{A}}{2} \right)^3$$

Space for Rough Work

Handwritten calculations for question 16:

$$V_Q = \frac{4}{3} \pi \frac{\sqrt{A}}{2}^3 \rho$$

$$M_Q = \frac{4}{3} \pi \left( \frac{\sqrt{A}}{2} \right)^3 \rho$$



$$\frac{4x + 0^3}{6} = 12$$

**PHYSICS**

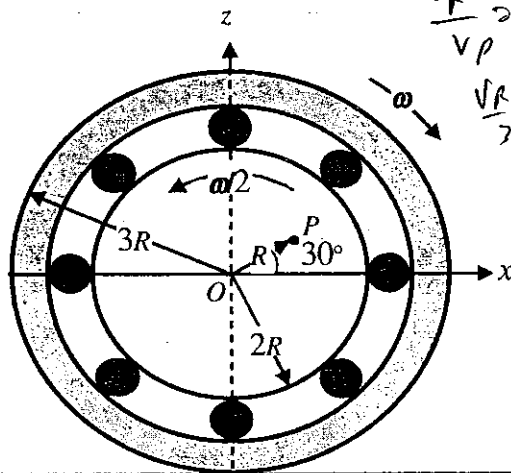
**PHYSICS**

17. Two spherical planets  $P$  and  $Q$  have the same uniform density  $\rho$ , masses  $M_P$  and  $M_Q$ , and surface areas  $A$  and  $4A$ , respectively. A spherical planet  $R$  also has uniform density  $\rho$  and its mass is  $(M_P + M_Q)$ . The escape velocities from the planets  $P$ ,  $Q$  and  $R$ , are  $V_P$ ,  $V_Q$  and  $V_R$ , respectively. Then

- (A)  $V_Q > V_R > V_P$     (B)  $V_R > V_Q > V_P$     (C)  $V_R/V_P = 3$     (D)  $V_P/V_Q = \frac{1}{2}$

18. The figure shows a system consisting of (i) a ring of outer radius  $3R$  rolling clockwise without slipping on a horizontal surface with angular speed  $\omega$  and (ii) an inner disc of radius  $2R$  rotating anti-clockwise with angular speed  $\omega/2$ . The ring and disc are separated by frictionless ball bearings. The system is in the  $x-z$  plane. The point  $P$  on the inner disc is at a distance  $R$  from the origin, where  $OP$  makes an angle of  $30^\circ$  with the horizontal. Then with respect to the horizontal surface,

$4\pi R^2 = A$   
 $R^2 = \frac{A}{4\pi}$   
 $4\pi R^2 = 4A$   
 $R^2 = \frac{A}{\pi}$



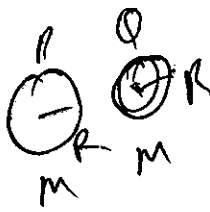
$\frac{V_R}{V_P} = 3$   
 $\frac{V_R}{V_P} = 3$   
 $V_P = \frac{1}{2} V_Q$   
 $V_P = \frac{V_R}{3}$   
 $\frac{V_Q}{V_P} = \frac{1}{2}$   
 $R_P = \frac{A}{4\pi}$      $R_Q = \frac{A}{\pi}$   
 $M_P$      $M_Q$   
 $A$      $4A$   
 $R$   
 $M_P + M_Q$

- (A) the point  $O$  has a linear velocity  $3R\omega\hat{i}$ .
- (B) the point  $P$  has a linear velocity  $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$ .
- (C) the point  $P$  has a linear velocity  $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$ .
- (D) the point  $P$  has a linear velocity  $\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\hat{i} + \frac{1}{4}R\omega\hat{k}$ .

$R_P = \sqrt{\frac{A}{4\pi}}$      $R_Q = \sqrt{\frac{A}{\pi}}$   
 $\frac{V_Q}{V_R} = \frac{2}{3}$   
 $V_Q > V_R$   
 $V_e^2 \propto \frac{M_{planet}}{R}$   
 $R_P < R_Q$

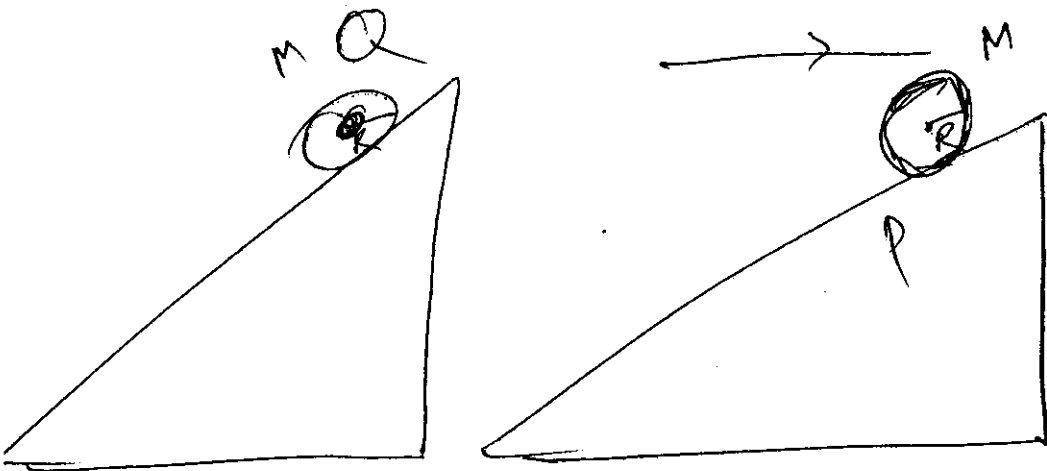
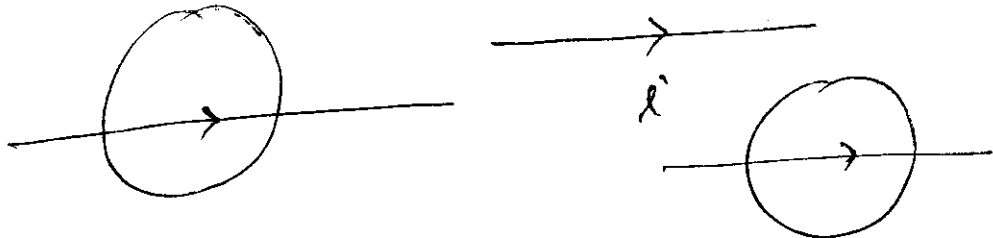
Space for Rough Work

$\frac{V_Q}{V_P} = \frac{1}{2}$   
 $\frac{4\pi \sqrt{\frac{A}{4\pi}}}{\sqrt{\frac{A}{\pi}}} = \frac{1}{2}$   
 $\frac{4\pi \cdot \frac{A}{4\pi}}{2 \cdot \frac{A}{4\pi}} = \frac{1}{2}$   
 $\frac{1}{2} = \frac{1}{2}$   
 $V_e^2 = \frac{2GM_P}{R}$   
 $V_P^2 = \frac{A}{3}$   
 $V_Q^2 = \frac{4A}{3}$   
 $V_Q > V_P$



19. Two solid cylinders  $P$  and  $Q$  of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder  $P$  has most of its mass concentrated near its surface, while  $Q$  has most of its mass concentrated near the axis. Which statement(s) is(are) correct ?
- (A) Both cylinders  $P$  and  $Q$  reach the ground at the same time.
  - (B) Cylinder  $P$  has larger linear acceleration than cylinder  $Q$ .
  - (C) Both cylinders reach the ground with same translational kinetic energy. ✓
  - (D) Cylinder  $Q$  reaches the ground with larger angular speed.
20. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are)
- ✓(A) The emf induced in the loop is zero if the current is constant.
  - (B) The emf induced in the loop is finite if the current is constant.
  - (C) The emf induced in the loop is zero if the current decreases at a steady rate.
  - ✓(D) The emf induced in the loop is finite if the current decreases at a steady rate.

Space for Rough Work



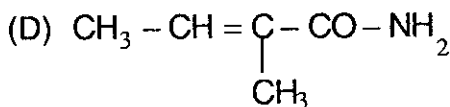
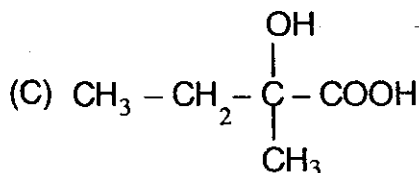
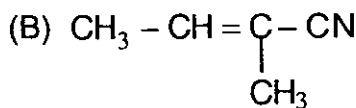
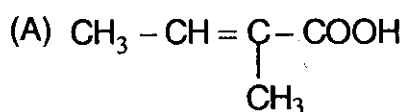
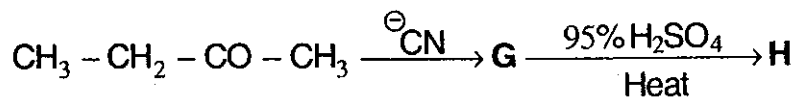


## PART II : CHEMISTRY

### SECTION I : Single Correct Answer Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. The major product H of the given reaction sequence is



22.  $\text{NiCl}_2\{\text{P}(\text{C}_2\text{H}_5)_2(\text{C}_6\text{H}_5)\}_2$  exhibits temperature dependent magnetic behaviour (paramagnetic / diamagnetic). The coordination geometries of  $\text{Ni}^{2+}$  in the paramagnetic and diamagnetic states are respectively

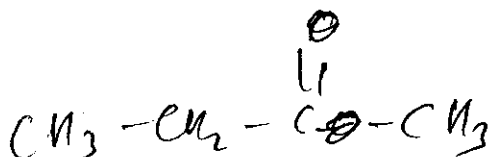
(A) tetrahedral and tetrahedral

(B) square planar and square planar

(C) tetrahedral and square planar

(D) square planar and tetrahedral

Space for Rough Work



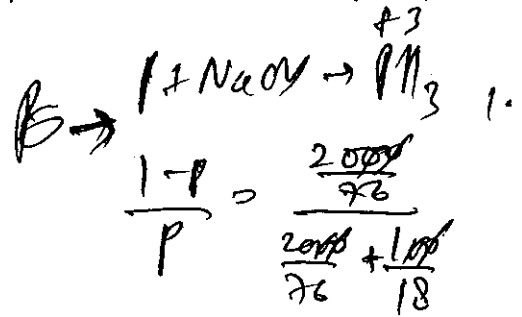


$$\frac{p^0 - p}{p^0} = X_{soln}$$

$$\frac{1-p}{p} = \frac{n_{sol} n_{tot}}{n_{sol} + n_{tot}}$$

23. In the cyanide extraction process of silver from argentite ore, the oxidizing and reducing agents used are
- (A) O<sub>2</sub> and CO respectively. (B) O<sub>2</sub> and Zn dust respectively.  
 (C) HNO<sub>3</sub> and Zn dust respectively. (D) HNO<sub>3</sub> and CO respectively.

24. The reaction of white phosphorus with aqueous NaOH gives phosphine along with another phosphorus containing compound. The reaction type; the oxidation states of phosphorus in phosphine and the other product are respectively
- (A) redox reaction; -3 and -5  
 (B) redox reaction; +3 and +5  
 (C) disproportionation reaction; -3 and +5  
 (D) disproportionation reaction; -3 and +3



25. The shape of XeO<sub>2</sub>F<sub>2</sub> molecule is

- (A) trigonal bipyramidal  
 (B) square planar  
 (C) tetrahedral  
 (D) see-saw

Space for Rough Work

Handwritten calculations for XeO<sub>2</sub>F<sub>2</sub> shape:

$$\frac{1-p}{p} = \frac{26}{76} = \frac{1+5}{38 \cdot 9}$$

$$\frac{1-p}{p} = \frac{26}{76} = \frac{208}{760}$$

$$9 + 190 - 9p - 190p = 9p$$

$$199 = 208p$$

$$p = \frac{199}{208}$$

Handwritten Lewis structure for XeO<sub>2</sub>F<sub>2</sub> showing a square planar geometry with two lone pairs on the Xe atom.

Handwritten orbital diagram for Xe (7s, 7p, 7d) and F (2s, 2p) showing electron configurations.



~~1120~~

$$m = \frac{n_{\text{sol}}}{0.1}$$

2.5g in 100g water  $\Delta T_b = 2^\circ\text{C}$

$$\Delta T_b = K_b m$$

$$2 = 0.76 m$$

$$m = \frac{2}{0.76} = \frac{200}{76}$$

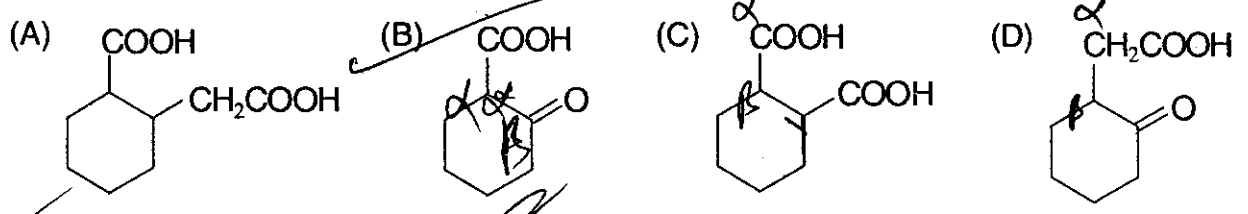
CHEMISTRY

26. For a dilute solution containing 2.5 g of a non-volatile non-electrolyte solute in 100 g of water, the elevation in boiling point at 1 atm pressure is  $2^\circ\text{C}$ . Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of the solution is (take  $K_b = 0.76 \text{ K kg mol}^{-1}$ )

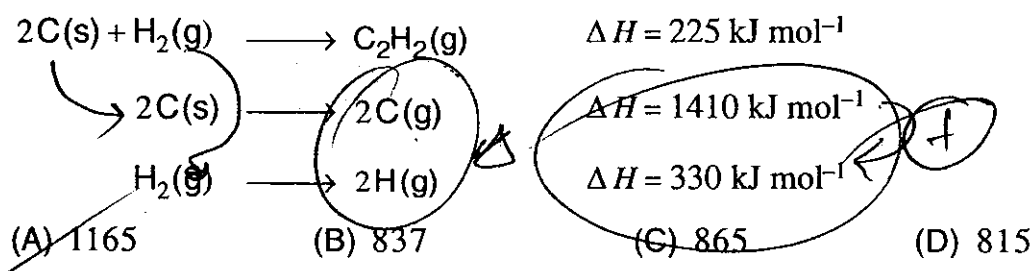
- (A) 724
- (B) 740
- (C) 736
- (D) 718

$$m = \frac{200}{76}$$

27. The compound that undergoes decarboxylation most readily under mild condition is



28. Using the data provided, calculate the multiple bond energy ( $\text{kJ mol}^{-1}$ ) of a  $\text{C}\equiv\text{C}$  bond in  $\text{C}_2\text{H}_2$ . That energy is (take the bond energy of a  $\text{C-H}$  bond as  $350 \text{ kJ mol}^{-1}$ .)



- (A) 1165
- (B) 837
- (C) 865
- (D) 815

$$\begin{array}{r} 1410 \\ 225 \\ \hline 1185 \end{array}$$

Space for Rough Work

$2\text{C} \rightarrow 2\text{C}$   
 $\text{H}_2 \rightarrow 2\text{H}$   
 $2\text{C} + \text{H}_2 \rightarrow$   
 $2\text{C(s)} + \text{H}_2(\text{g}) \rightarrow 2\text{C(g)} + 2\text{H(g)}$

$\Delta H =$   
 $1410$   
 $330$   
 $\hline 1740$

$\text{H}-\text{C}\equiv\text{C}-\text{H}$   
 $350 \quad 350$   
 $700$   
 $1740 - 700 = 1040$

$\alpha \beta \text{ Keto}$   $\beta \text{ Keto acid}$   
 $\alpha \text{ amino acid}$

$1965$   
 $- 700$   
 $\hline 1265$

$1410$   
 $330$   
 $\hline 1740$

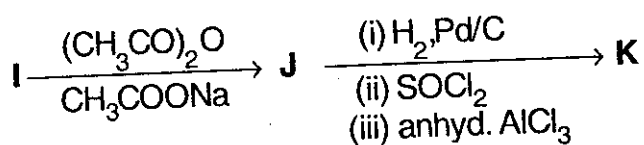


## SECTION II : Paragraph Type

This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

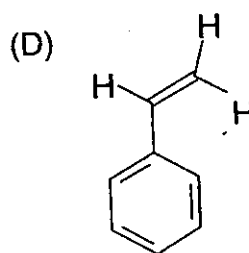
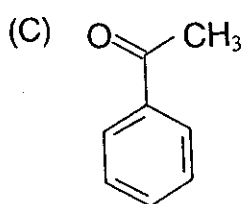
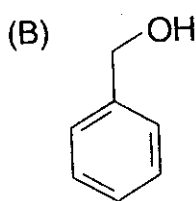
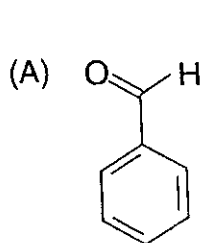
## Paragraph for Questions 29 and 30

In the following reaction sequence, the compound J is an intermediate.

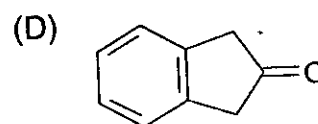
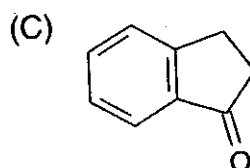
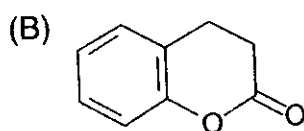
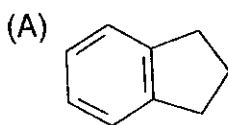


J ( $\text{C}_9\text{H}_8\text{O}_2$ ) gives effervescence on treatment with  $\text{NaHCO}_3$  and a positive Baeyer's test.

29. The compound I is



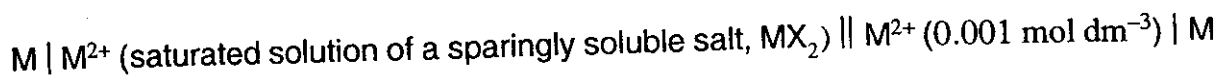
30. The compound K is



Space for Rough Work

Paragraph for Questions 31 and 32

The electrochemical cell shown below is a concentration cell.



The emf of the cell depends on the difference in concentrations of  $M^{2+}$  ions at the two electrodes.

The emf of the cell at 298 K is 0.059 V.

31. The value of  $\Delta G$  ( $\text{kJ mol}^{-1}$ ) for the given cell is (take  $1F = 96500 \text{ C mol}^{-1}$ )

- (A) -5.7                      (B) 5.7                      (C) 11.4                      (D) -11.4

32. The solubility product ( $K_{sp}$ ;  $\text{mol}^3 \text{ dm}^{-9}$ ) of  $MX_2$  at 298 K based on the information available for the given concentration cell is (take  $2.303 \times R \times 298/F = 0.059 \text{ V}$ )

- (A)  $1 \times 10^{-15}$                       (B)  $4 \times 10^{-15}$                       (C)  $1 \times 10^{-12}$                       (D)  $4 \times 10^{-12}$

Space for Rough Work

$n$

0.059 V

$$W = V \cdot Q$$

$$W = E \cdot q$$

$$= E \cdot e \cdot N_A$$

$$\Delta G = n F E$$

$$= 2 \times 96500 \times 0.059$$

$$2 \times 965 \times 5.9$$

$$\frac{1930}{2} \quad \frac{193}{59}$$

$$\begin{array}{r} 965 \\ 193 \\ \hline 1930 \\ 1965 \\ \hline 17387 \end{array}$$



## Paragraph for Questions 33 and 34

Bleaching powder and bleach solution are produced on a large scale and used in several household products. The effectiveness of bleach solution is often measured by iodometry.

33. Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is
- (A)  $\text{Cl}_2\text{O}$                       (B)  $\text{Cl}_2\text{O}_7$                       (C)  $\text{ClO}_2$                       (D)  $\text{Cl}_2\text{O}_6$
34. 25 mL of household bleach solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4 N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N  $\text{Na}_2\text{S}_2\text{O}_3$  was used to reach the end point. The molarity of the household bleach solution is
- (A) 0.48 M                      (B) 0.96 M                      (C) 0.24 M                      (D) 0.024 M

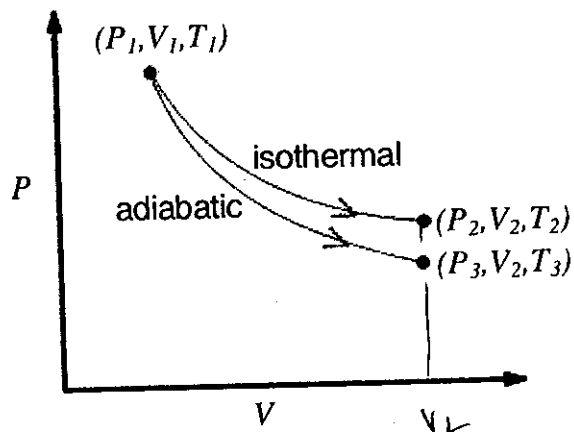
Space for Rough Work

25 mL Bleach      30 mL 0.5 M KI      10 mL 4 N acetic acid

SECTION III : Multiple Correct Answer(s) Type

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

35. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct?



$T_1 = T_2 > T_3$

(A)  $T_1 = T_2$

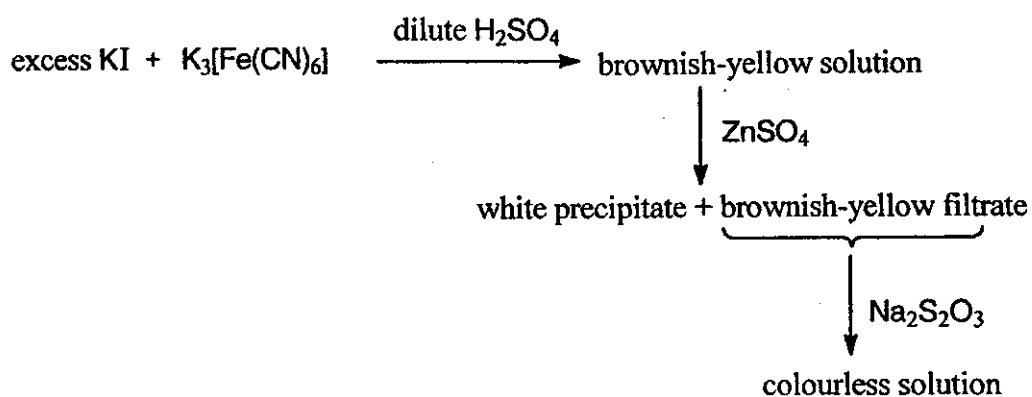
(C)  $w_{\text{isothermal}} > w_{\text{adiabatic}}$

(B)  $T_3 > T_1$

(D)  $\Delta U_{\text{isothermal}} > \Delta U_{\text{adiabatic}}$

Space for Rough Work

36. For the given aqueous reactions, which of the statement(s) is (are) true ?



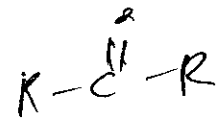
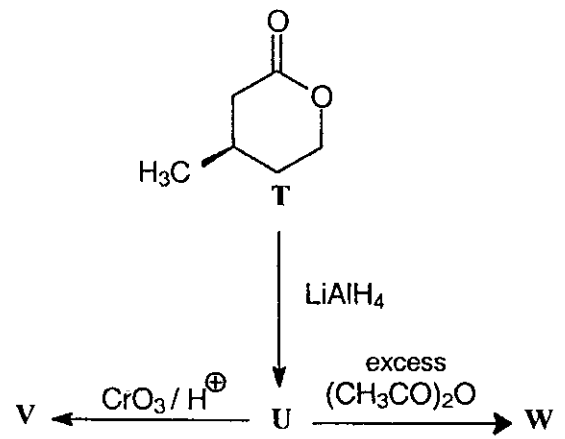
- (A) The first reaction is a redox reaction.  
 (B) White precipitate is  $\text{Zn}_3[\text{Fe}(\text{CN})_6]_2$ .  
 (C) Addition of filtrate to starch solution gives blue colour.  
 (D) White precipitate is soluble in NaOH solution.

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Space for Rough Work



37. With reference to the scheme given, which of the given statement(s) about T, U, V and W is (are) correct ?

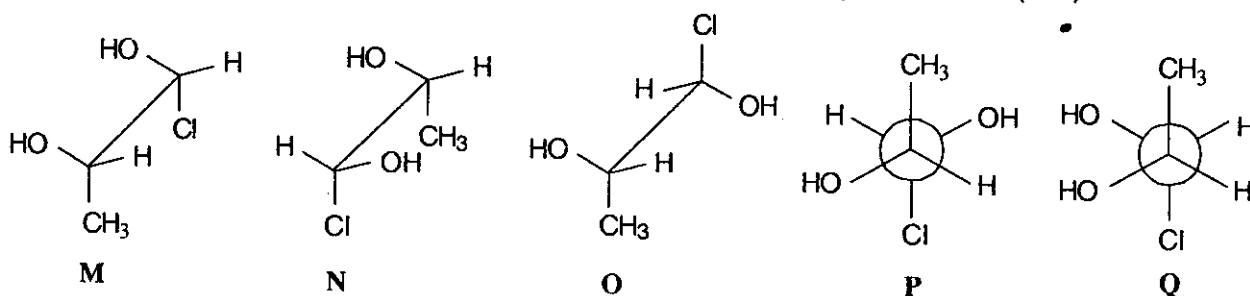


- (A) T is soluble in hot aqueous NaOH
- (B) U is optically active
- (C) Molecular formula of W is  $\text{C}_{10}\text{H}_{18}\text{O}_4$
- (D) V gives effervescence on treatment with aqueous  $\text{NaHCO}_3$

Space for Rough Work



38. Which of the given statement(s) about N, O, P and Q with respect to M is (are) correct ?



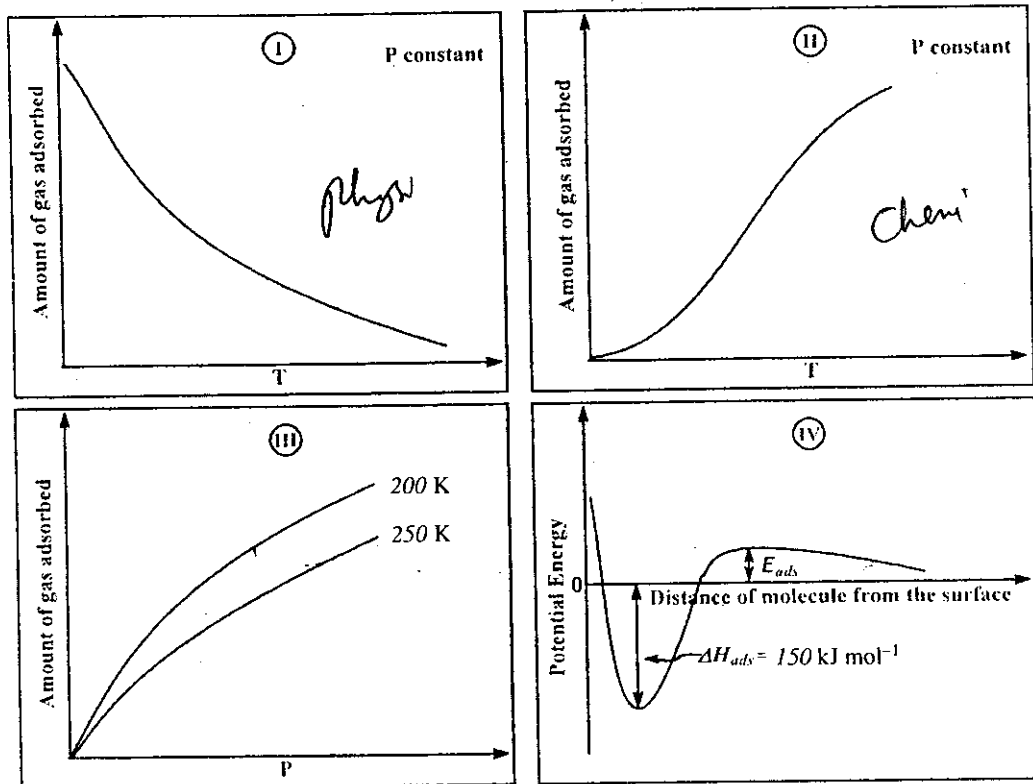
- (A) M and N are non-mirror image stereoisomers ✓  
(B) M and O are identical ✓  
(C) M and P are enantiomers  
(D) M and Q are identical ✓
39. With respect to graphite and diamond, which of the statement(s) given below is (are) correct ?
- (A) Graphite is harder than diamond. ✗  
(B) Graphite has higher electrical conductivity than diamond. ✓  
(C) Graphite has higher thermal conductivity than diamond. ✓  
(D) Graphite has higher C-C bond order than diamond. ✗

Space for Rough Work





40. The given graphs / data I, II, III and IV represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice(s) about I, II, III and IV is (are) correct?



- (A) I is physisorption and II is chemisorption ✓  
(B) I is physisorption and III is chemisorption ✗  
(C) IV is chemisorption and II is chemisorption  
(D) IV is chemisorption and III is chemisorption ✗

Space for Rough Work



# PART III : MATHEMATICS

$$\frac{1}{5} \rightarrow \frac{1}{2x}$$

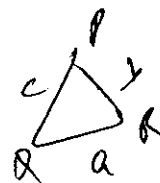
## SECTION I : Single Correct Answer Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

41. Let  $a_1, a_2, a_3, \dots$  be in harmonic progression with  $a_1 = 5$  and  $a_{20} = 25$ . The least positive integer  $n$  for which  $a_n < 0$  is  
 (A) 22 (B) 23 (C) 24 (D) 25
42. The equation of a plane passing through the line of intersection of the planes  $x + 2y + 3z = 2$  and  $x - y + z = 3$  and at a distance  $\frac{2}{\sqrt{3}}$  from the point  $(3, 1, -1)$  is  
 (A)  $5x - 11y + z = 17$  (B)  $\sqrt{2}x + y = 3\sqrt{2} - 1$   
 (C)  $x + y + z = \sqrt{3}$  (D)  $x - \sqrt{2}y = 1 - \sqrt{2}$
43. Let  $PQR$  be a triangle of area  $\Delta$  with  $a = 2, b = \frac{7}{2}$  and  $c = \frac{5}{2}$ , where  $a, b$  and  $c$  are the lengths of the sides of the triangle opposite to the angles at  $P, Q$  and  $R$  respectively.

Then  $\frac{2 \sin P - \sin 2P}{2 \sin P + \sin 2P}$  equals

- (A)  $\frac{3}{4\Delta}$  (B)  $\frac{45}{4\Delta}$  (C)  $\left(\frac{3}{4\Delta}\right)^2$  (D)  $\left(\frac{45}{4\Delta}\right)^2$



Space for Rough Work

$$a_1 = 5$$

$$\frac{(1+a)^{1/6} - 1}{(1+a)^{1/3}}$$

$$a_1 \quad a_2 \quad a_3 \quad \dots \quad a_n$$

$$\frac{1}{a_1} \quad \frac{1}{a_2} \quad \frac{1}{a_3} \quad \dots \quad \frac{1}{a_n}$$

$$T_1 = \frac{1}{5}$$

$$\frac{1}{a_n} = \frac{1}{5} + \frac{(n-1)4}{5 \times 19}$$

$$T_{20} = \frac{1}{25}$$

$$\frac{5}{a_n} = \frac{19(1-n)4}{19}$$

$$\frac{19 \times 5}{a_n} = 19 + 4 - 4n$$

$$= 23 - 4n$$

$$T_n = a + (n-1)d$$

$$T_{20} = \frac{1}{25} + (19)d$$

$$\frac{1}{25} = \frac{1}{5} + (19)d$$

$$\frac{1}{25} - \frac{1}{5} = 19d$$

$$T_n = \frac{1}{5} + \frac{(n-1)4}{19}$$



$$\alpha(a) = \frac{(1+a)^{1/2} - 1}{(1+a)^{3/2} - 1} = \frac{\frac{1}{2}(1+a)^{-1/2}}{\frac{3}{2}(1+a)^{-3/2}} = \frac{1}{3} \frac{1}{(1+a)}$$

44. If  $\vec{a}$  and  $\vec{b}$  are vectors such that  $|\vec{a} + \vec{b}| = \sqrt{29}$  and  $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$ ,

then a possible value of  $(\vec{a} + \vec{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$  is

- (A) 0 (B) 3 (C) 4 (D) 8

$$\frac{(1+a)^{1/2} - 1}{(1+a)^{3/2} - 1}$$

45. If  $P$  is a  $3 \times 3$  matrix such that  $P^T = 2P + I$ , where  $P^T$  is the transpose of  $P$  and  $I$  is the  $3 \times 3$

identity matrix, then there exists a column matrix  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  such that

- (A)  $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  (B)  $PX = X$  (C)  $PX = 2X$  (D)  $PX = -X$

$$\frac{\frac{1}{2}(1+a)^{-1/2}}{\frac{3}{2}(1+a)^{-3/2}} = \frac{1}{3(1+a)}$$

46. Let  $\alpha(a)$  and  $\beta(a)$  be the roots of the equation

$$\frac{(\sqrt[3]{1+a}-1)}{a}x^2 + \frac{(\sqrt{1+a}-1)}{b}x + \frac{(\sqrt[6]{1+a}-1)}{c} = 0 \text{ where } a > -1. \alpha(a) + \beta(a) =$$

Then  $\lim_{a \rightarrow 0^+} \alpha(a)$  and  $\lim_{a \rightarrow 0^+} \beta(a)$  are

- (A)  $-\frac{5}{2}$  and 1 (B)  $-\frac{1}{2}$  and -1 (C)  $-\frac{7}{2}$  and 2 (D)  $-\frac{9}{2}$  and 3

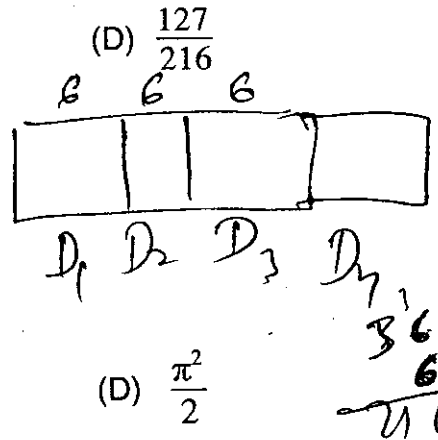
47. Four fair dice  $D_1, D_2, D_3$  and  $D_4$ , each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that  $D_4$  shows a number appearing on one of  $D_1, D_2$  and  $D_3$  is

- (A)  $\frac{91}{216}$  (B)  $\frac{108}{216}$  (C)  $\frac{125}{216}$  (D)  $\frac{127}{216}$

48. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \left( x^2 + \ln \frac{\pi+x}{\pi-x} \right) \cos x \, dx \text{ is}$$

- (A) 0 (B)  $\frac{\pi^2}{2} - 4$  (C)  $\frac{\pi^2}{2} + 4$  (D)  $\frac{\pi^2}{2}$



Space for Rough Work

$$\frac{1}{2x} - \frac{1}{2} = \frac{1}{2x} - \frac{x}{2x} = \frac{1-x}{2x}$$



SECTION II : Paragraph Type

This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 49 and 50

A tangent PT is drawn to the circle  $x^2 + y^2 = 4$  at the point  $P(\sqrt{3}, 1)$ . A straight line L, perpendicular to PT is a tangent to the circle  $(x-3)^2 + y^2 = 1$ .

49. A possible equation of L is

- (A)  $x - \sqrt{3}y = 1$       (B)  $x + \sqrt{3}y = 1$       (C)  $x - \sqrt{3}y = -1$       (D)  $x + \sqrt{3}y = 5$

50. A common tangent of the two circles is

- (A)  $x = 4$       (B)  $y = 2$       (C)  $x + \sqrt{3}y = 4$       (D)  $x + 2\sqrt{2}y = 6$

Space for Rough Work

Handwritten work for question 49:

$$x^2 + y^2 = 4 + x^2 + 9 - 6x + y^2 = 1$$

$$-4 - 9 = 6x$$

$$-13 = 6x$$

$$x = -13/6$$

Handwritten work for question 50:

$$\frac{|-4|}{\sqrt{4}} = \frac{2}{2} = 1$$

$$\frac{|2-1|}{\sqrt{1}} = 1$$

Handwritten work for question 50:

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2x}{2y} = -x/y = -\sqrt{3}/1 = -\sqrt{3}$$

Handwritten notes:

$$m_1, m_2 = -1$$

$$m_2 = \sqrt{3}$$

A circled answer  $\frac{1}{\sqrt{3}}$  is shown.



$$\left[ \frac{2(x-1)}{x+1} - \ln x \right] (1-x)^2 \ln^2 x + x^2$$

**Paragraph for Questions 51 and 52**

Let  $f(x) = (1-x)^2 \sin^2 x + x^2$  for all  $x \in \mathbb{R}$ , and let  $g(x) = \int_1^x \left( \frac{2(t-1)}{t+1} - \ln t \right) f(t) dt$  for all  $x \in (1, \infty)$ .

51. Consider the statements :

**P** : There exists some  $x \in \mathbb{R}$  such that  $f(x) + 2x = 2(1+x^2)$

**Q** : There exists some  $x \in \mathbb{R}$  such that  $2f(x) + 1 = 2x(1+x)$

$$f(x) = 2(1+x^2) - 2x$$

$$2(1+x^2) - 2x = (1-x)^2$$

Then

(A) both **P** and **Q** are true

(B) **P** is true and **Q** is false

(C) **P** is false and **Q** is true

(D) both **P** and **Q** are false

52. Which of the following is true?

(A)  $g$  is increasing on  $(1, \infty)$

(B)  $g$  is decreasing on  $(1, \infty)$

(C)  $g$  is increasing on  $(1, 2)$  and decreasing on  $(2, \infty)$

(D)  $g$  is decreasing on  $(1, 2)$  and increasing on  $(2, \infty)$

$$(1-x^2) \ln^2 x + x^2 = 2(1+x^2) - 2x$$

$$\ln^2 x - x^2 \ln^2 x + x^2 = 2 + 2x^2 - 2x$$

$$\ln^2 x + x^2(1 - \ln^2 x) = 2(1+x^2 - x)$$

$$g'(x) = \left[ \frac{2(x-1)}{x+1} - \ln x \right] f(x)$$

Space for Rough Work

$$m = \frac{1}{\sqrt{3}} \quad y = \frac{1}{\sqrt{3}}x + c$$

$$\sqrt{3}y - x - \sqrt{3}c = 0 \quad \frac{|\sqrt{3}y - x - \sqrt{3}c|}{\sqrt{3+1}} = 1$$

$$3 + \sqrt{3}c = 2$$

$$\sqrt{3}c = -1$$

$$c = -\frac{1}{\sqrt{3}}$$

$$y = \frac{1}{\sqrt{3}}x - \frac{1}{\sqrt{3}}$$

$$\sqrt{3}y - x = -1$$

$$x - \sqrt{3}y = 1$$

$$x - \sqrt{3}y - 1 = 0$$

$$\frac{3-1}{\sqrt{4}} = 1$$

$$\frac{2}{2}$$



Paragraph for Questions 53 and 54

Let  $a_n$  denote the number of all  $n$ -digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let  $b_n =$  the number of such  $n$ -digit integers ending with digit 1 and  $c_n =$  the number of such  $n$ -digit integers ending with digit 0.

53. The value of  $b_6$  is

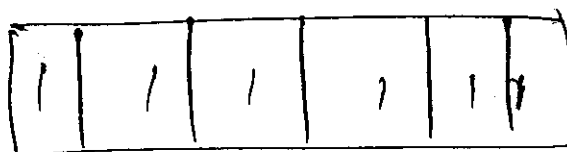
- (A) 7
- (B) 8
- (C) 9
- (D) 11

54. Which of the following is correct?

- (A)  $a_{17} = a_{16} + a_{15}$
- (B)  $c_{17} \neq c_{16} + c_{15}$
- (C)  $b_{17} \neq b_{16} + c_{16}$
- (D)  $a_{17} = c_{17} + b_{16}$

Space for Rough Work

$n = 6$



$111111$  ✓  
 $101111$  ✓       $101011$  ✓  
 $110111$  ✓       $101101$  ✓  
 $111101$  ✓       $110101$  ✓  
 $111011$  ✓  
~~111101~~  
~~111011~~  
~~110101~~  
~~101011~~  
~~101101~~  
~~101011~~  
~~101101~~

**SECTION III : Multiple Correct Answer(s) Type**

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

55. For every integer  $n$ , let  $a_n$  and  $b_n$  be real numbers. Let function  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}, \text{ for all integers } n.$$

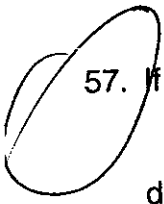
If  $f$  is continuous, then which of the following hold(s) for all  $n$ ?

- (A)  $a_{n-1} - b_{n-1} = 0$     (B)  $a_n - b_n = 1$     (C)  $a_n - b_{n+1} = 1$     (D)  $a_{n-1} - b_n = -1$

56. If the straight lines  $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$  and  $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$  are coplanar, then the plane(s) containing these two lines is(are)

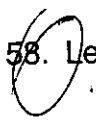
- (A)  $y + 2z = -1$     (B)  $y + z = -1$     (C)  $y - z = -1$     (D)  $y - 2z = -1$

Space for Rough Work



57. If the adjoint of a 3x3 matrix P is  $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$ , then the possible value(s) of the determinant of P is (are)

- (A) -2 (B) -1 (C) 1 (D) 2



58. Let  $f: (-1, 1) \rightarrow \mathbb{R}$  be such that  $f(\cos 4\theta) = \frac{2}{2 - \sec^2 \theta}$  for  $\theta \in (0, \frac{\pi}{4}) \cup (\frac{\pi}{4}, \frac{\pi}{2})$ . Then the value(s) of  $f(\frac{1}{3})$  is (are)

- (A)  $1 - \sqrt{\frac{3}{2}}$  (B)  $1 + \sqrt{\frac{3}{2}}$  (C)  $1 - \sqrt{\frac{2}{3}}$  (D)  $1 + \sqrt{\frac{2}{3}}$

Space for Rough Work

$\cos 4\theta = \frac{1}{3}$

$f'(x) = e^{x^2} (x-2)(x-3)$

$f''(x) = 2xe^{x^2} (x-2)(x-3) + e^{x^2} (x-3) + e^{x^2} (x-2)$

$f''(2) = 4e^4 e^4 (-1) = -4e^4$

$f''(3) = e^9 = e^9$

adj = Tr of cofactor

Req

Cofactor =  $\begin{bmatrix} 1 & 2 & 1 \\ 4 & 1 & 7 \\ 4 & 7 & 3 \end{bmatrix}$

Min =  $\begin{bmatrix} 1 & -2 & 1 \\ -4 & 1 & -1 \\ 4 & -7 & 3 \end{bmatrix}$

$\frac{1}{2} + \frac{1}{3} - \frac{1}{1}$

$\frac{3+2-6}{6} = \frac{-1}{6}$

Handwritten table with numbers 1, 2, 3 in a grid structure.



59. Let  $X$  and  $Y$  be two events such that  $P(X|Y) = \frac{1}{2}$ ,  $P(Y|X) = \frac{1}{3}$  and  $P(X \cap Y) = \frac{1}{6}$ .

Which of the following is (are) correct?

~~(A)  $P(X \cup Y) = \frac{2}{3}$~~

(B)  $X$  and  $Y$  are independent ✓

(C)  $X$  and  $Y$  are not independent

~~(D)  $P(X^c \cap Y) = \frac{1}{3}$~~

60. If  $f(x) = \int_0^x e^{t^2} (t-2)(t-3) dt$  for all  $x \in (0, \infty)$ , then

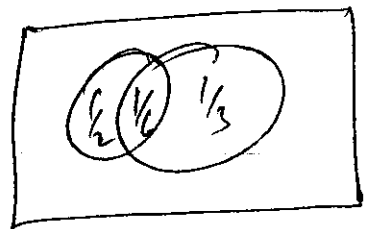
(A)  $f$  has a local maximum at  $x = 2$  ✓

(B)  $f$  is decreasing on  $(2, 3)$  ✓

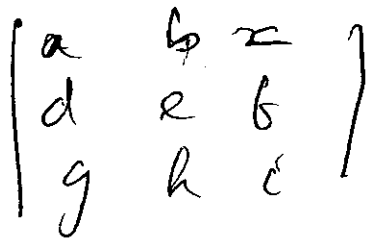
(C) there exists some  $c \in (0, \infty)$  such that  $f''(c) = 0$  ✗

(D)  $f$  has a local minimum at  $x = 3$  ✓

$P(\bar{X} \cap Y)$



Space for Rough Work



$P(X|Y) = \frac{1}{2}$

$P(X \cap Y) = \frac{1}{6}$

$\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$

$\frac{P(X \cap Y)}{P(X)} = \frac{1}{3}$

$\frac{1/6}{1/3} = \frac{1}{2}$

~~$P(X \cap Y)$~~

$P(X \cap Y) = \frac{1}{6}$

$P(Y) = \frac{1}{3}$

$P(X) = \frac{1}{2}$

$P(X) \cdot P(Y) = P(X \cap Y)$

$\frac{1/6}{1/3} = \frac{1}{2}$

$\frac{1/6}{1/2} = \frac{1}{3}$

